

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 6

Although the problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Step 1: I begin by translating some of the most important words into an equation.

"...the **average** number of colony collapses due to pesticides from 2007 to 2012 **compared** to the average from 2002 to 2007 **was** $1\frac{2}{3}$ **times** the **ratio of the averages** of incidents investigated during the same corresponding sets of years."

$$\frac{\text{aver. attr. to pesticides 2007 to 2012}}{\text{aver. attr. to pesticides 2002 to 2007}} = 1\frac{2}{3} * \frac{\text{aver. incidents inv. 2007 to 2012}}{\text{aver. incidents inv. 2002 to 2007}}$$

Step 2: Next, I look at the graphs and find the correct numbers to substitute into my equation.

The missing points were said to be "**the same**" so I assign them each the variable y .

$$\frac{\left(\frac{2 + 5 + 9 + 14 + y + y}{6}\right)}{\left(\frac{5 + 8 + 4 + 1 + 2 + 2}{6}\right)} = 1\frac{2}{3} * \frac{\left(\frac{24 + 39 + 42 + 27 + 18 + 30}{6}\right)}{\left(\frac{30 + 27 + 33 + 21 + 15 + 24}{6}\right)}$$

Step 3: I simplify and solve, first adding up the numbers in the parentheses, then performing the rest of the calculations.

$$\frac{\left(\frac{30 + 2y}{6}\right)}{\left(\frac{22}{6}\right)} = 1\frac{2}{3} * \frac{\left(\frac{180}{6}\right)}{\left(\frac{150}{6}\right)}$$

$$\left(\frac{30 + 2y}{6}\right) * \left(\frac{6}{22}\right) = \frac{5}{3} * \frac{30}{25}$$

$$\frac{6(30 + 2y)}{132} = \frac{5}{3} * \frac{6}{5}$$

$$\frac{180 + 12y}{132} = 2$$

$$\frac{180 + 12y}{132} = \frac{2}{1}$$

$$1(180 + 12y) = 2(132)$$

$$180 + 12y = 264$$

$$12y = 84$$

$$y = 7$$

Now I know the missing numbers. Each one is 7!

Step 4: I find the total number of bee colony collapses due to pesticides, for 2002 to 2012, by adding up all the numbers in that set.

$$\text{Total} = 5 + 8 + 4 + 1 + 2 + 2 + 5 + 9 + 14 + 7 + 7$$

$$\text{Total} = 64$$

Answer: The total number of bee colony collapses due to pesticides in County A, North Dakota from 2002 to 2012 was **64**.

THE HARDEST MATH PROBLEM

GRADE 7

The leaders of the EARTH Club learned a lot from the research they collected about bees. Now they are making plans to turn the facts into action.


"I bet a lot of students don't know that honeybees pollinate about one-third of the nation's crops, but are disappearing at an alarming rate," says Maria.

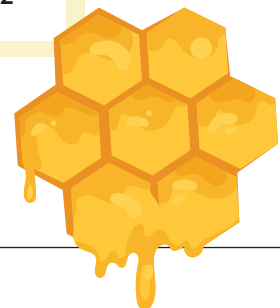
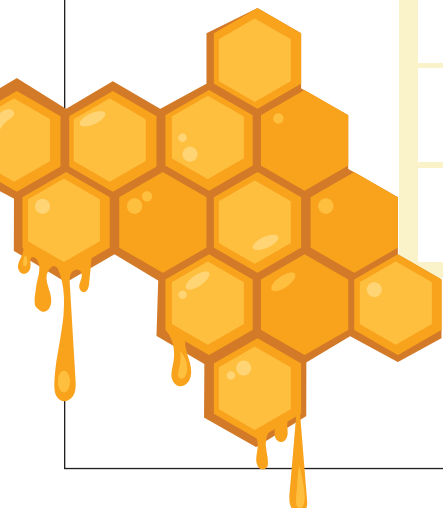
"Yeah, and pesticides are a big part of the problem," adds Vishal. "The bees need our help!"

The EARTH Club decides to launch a few exciting projects at their school to create buzz about saving the bees. It's a lot of work for the EARTH Club, and they're counting on math help from their newest member—you!

Solve the Problem

The 7th graders are planting a school garden and have decided to use only organic pesticides. They considered six products with similar ingredients that are safe for bees. Jade just finished placing their orders when—yikes!—her laptop died! And she can't remember the total amount she spent on the three different bottles she ordered. Of two combinations that had the same median for increase in honey production, she ordered the one with the lower average cost per ounce and was able to stay within the club budget of \$120. **How much money did they spend?**

Organic Pesticides 			
Product	% increase in honey production (per year)	Bulk cost (shipped in 64 oz bottles)	Shipping cost
MintMix	62%	\$34.29/ qt	\$1.15/ half-gallon
ZenEarthinol	58%	\$88.50/ gal	\$2.50/ bottle
Mito-Down	71%	\$130.60/ 2.5 gal	\$3.50/ gallon
VarroAway	99%	\$124.80/ 1.5 gal	5% of subtotal
Garden+	95%	\$75.16/ gal	Free
NoPest	80%	\$223/ 2 gal	\$.05/oz



THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 7

Although the problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Step 1: I begin by calculating the cost of 1 bottle of each product.

Each bottle contains 64 oz. I will convert gallons (the bulk cost given) to ounces to find the per-bottle cost.

$1 \text{ gal} * \frac{4 \text{ qt}}{1 \text{ gal}} * \frac{2 \text{ pints}}{1 \text{ qt}} * \frac{2 \text{ cups}}{1 \text{ pint}} * \frac{8 \text{ oz}}{1 \text{ cup}} = 128 \text{ oz.}$	1 gallon = 128 oz.	Each bottle is $\frac{1}{2}$ gallon
	Product	Shipping Cost
MintMix:	$\frac{\$34.29}{1 \text{ gal}} * \frac{4 \text{ qt}}{1 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$68.58}{1 \text{ bottle}}$	$\$1.15$ $\$68.58 + \$1.15 = \$69.73$
ZenEarthinol:	$\frac{\$88.50}{1 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$44.25}{1 \text{ bottle}}$	$\$2.50$ $\$44.25 + \$2.50 = \$46.75$
Mito-Down:	$\frac{\$130.60}{2.5 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$26.12}{1 \text{ bottle}}$	$\$1.75$ $\$26.12 + \$1.75 = \$27.87$
VarroAway:	$\frac{\$124.80}{1.5 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$41.60}{1 \text{ bottle}}$	$(0.05)(\$41.60) = \2.08 $\$41.60 + \$2.08 = \$43.68$
Garden+:	$\frac{\$75.16}{1 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$37.58}{1 \text{ bottle}}$	Free $\$37.58$
NoPest:	$\frac{\$223}{2 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$55.75}{1 \text{ bottle}}$	$(\$0.05)(64) = \3.20 $\$55.75 + \$3.20 = \$58.95$

Step 2: Next, I find the combinations of 3 different bottles that have a total cost less than or equal to the budget amount of \$120.

I consider the most expensive bottle, MintMix, at a price of \$69.73. I subtract it from the club's budget:

$$\$120.00 - \$69.73 = \$50.27$$

The sum of the two remaining bottles within this set must be less than or equal to \$50.27 to stay within budget. If I subtract the lowest priced bottle ($\$50.27 - \$27.87 = \$22.40$), I find there is not enough money left for any of the others to be the third bottle.

So, I eliminate this bottle, MintMix, as being a possibility in the final set of 3.

I use the same set of steps and reasoning to consider the next most expensive bottle, NoPest, which costs \$58.95.

$$\$120.00 - \$58.95 = \$61.05$$

$$\$61.05 - \$27.87 = \$33.18$$

I cannot get a third bottle for less than or equal to \$33.18, so I eliminate NoPest as a possibility, too.

(continued on next page)

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 7

Although each problem does have a correct numeric solution, there are multiple pathways students can take to arrive at the answer.

(continued from previous page)

There are only 3 possibilities that will give a total less than or equal to the club budget of \$120.

$$\$46.75 + \$27.87 + \$43.68 = \$118.30 \quad (\text{Zen / Mito / VarroA})$$

$$\$46.75 + \$27.87 + \$37.58 = \$112.20 \quad (\text{Zen / Mito / Garden+})$$

$$\$27.87 + \$43.68 + \$37.58 = \$109.13 \quad (\text{Mito / VarroA / Garden+})$$

Step 3: Now, regarding the two combinations with the same median increase in honey production, I must find the one with the lower cost per ounce.

The median of a data set is the middle value when the data is arranged in ascending order (from low to high).

I organize the percent increases in honey production:

$$\text{Zen/Mito/VarroA} \quad 58\%, 71\%, 99\% \quad \text{median} = 71\%$$

$$\text{Zen/Mito/Garden+} \quad 58\%, 71\%, 95\% \quad \text{median} = 71\%$$

$$\text{Mito/VarroA/Garden+} \quad 71\%, 95\%, 99\% \quad \text{median} = 95\%$$

Since the first two have the same median, I have to find the one that has "the lowest average cost per ounce."

To find the average cost per ounce, I will find each product's cost per ounce, then average those 3 unit rates.

$$\text{Zen/Mito/VarroA} \quad \frac{\left(\frac{\$46.75}{64}\right) + \left(\frac{\$27.87}{64}\right) + \left(\frac{\$43.68}{64}\right)}{3} = \frac{\$1.8484375}{3} = \$0.6161458... \approx \$0.62$$

$$\text{Zen/Mito/Garden+} \quad \frac{\left(\frac{\$46.75}{64}\right) + \left(\frac{\$27.87}{64}\right) + \left(\frac{\$37.58}{64}\right)}{3} = \frac{\$1.753125}{3} = \$0.584375 \approx \$0.58$$

I find that the combination of Zen/Mito/Garden+ has the lower unit cost per ounce.

I could also have skipped calculating cost per ounce since there is only 1 bottle that is different in the combinations. One has VarroA and the other Garden+. Since both bottles contain 64 oz and the Garden+ bottle costs less than the VarroA, I know the average cost per ounce will be less in the combination with Garden+.

Step 4: Looking back at my previous calculations in step 2, I see that the total cost of 1 bottle each of ZenEarthinol, Mito-Down, and Garden+ is \$112.20.

Answer: The 7th graders spent a total of **\$112.20** on bee-friendly pesticides.

THE HARDEST MATH PROBLEM

GRADE 8

The leaders of the EARTH Club learned a lot from the research they collected about bees. Now they are making plans to turn the facts into action.

"I bet a lot of students don't know that honeybees pollinate about one-third of the nation's crops, but are disappearing at an alarming rate," says Maria.

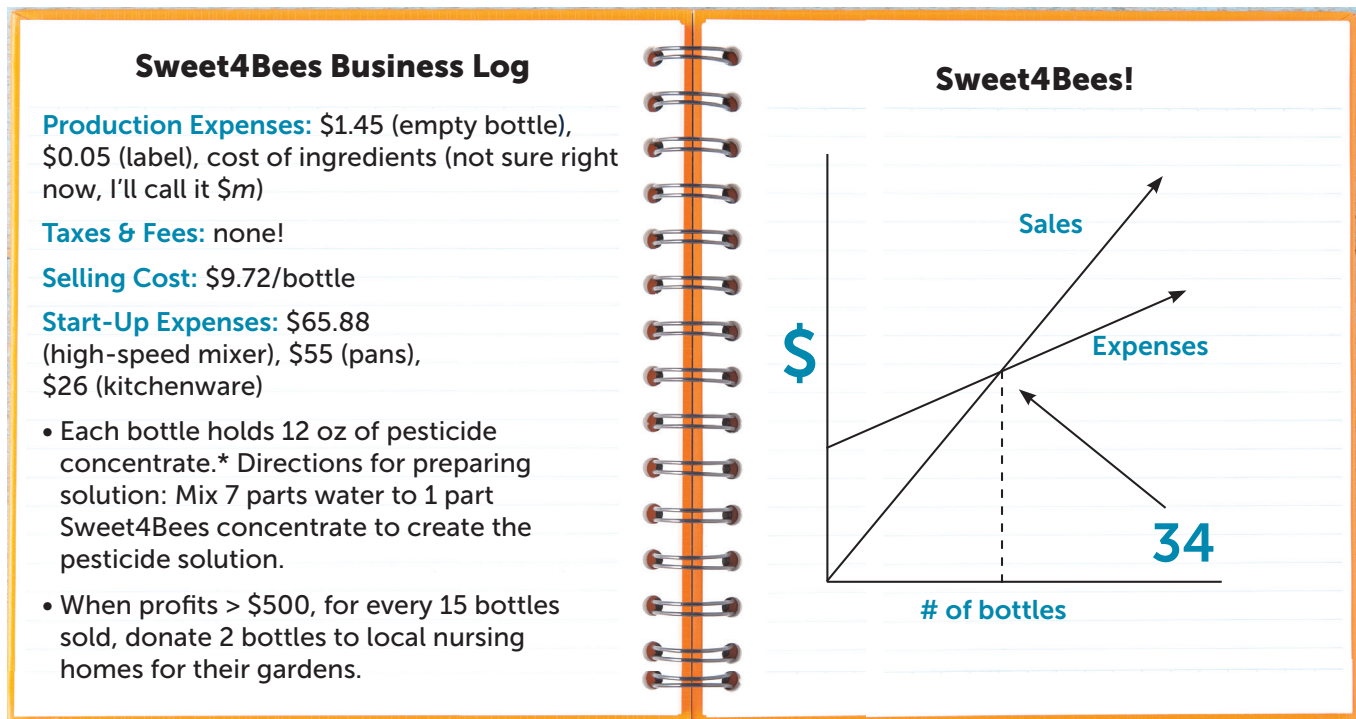
"Yeah, and pesticides are a big part of the problem," adds Vishal. "The bees need our help!"

The EARTH Club decides to launch a few exciting projects at their school to create buzz about saving the bees. It's a lot of work for the EARTH Club, and they're counting on math help from their newest member—you!

Solve the Problem

The 8th graders are volunteering to make and sell organic pesticides, called Sweet4Bees, that will protect plants in local gardens without harming bees. The teens also plan to donate some of their product to local nursing homes to use in their gardens. Jade is so excited! She's already started a business log in her notebook, including a graph with a break-even point of 34 bottles sold.

Suppose all Jade's notes become true. **What is the club's overall profit per ounce of pesticide solution, when 26 bottles have been donated to the nursing homes? Round to the nearest cent.**



*concentrate = a stronger version of a liquid product, intended to be diluted with water to make its final form

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 8

Although each problem does have a correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Step 1: I need to find the “total overall profit,” so I start with the profit equation.

$$\text{Profit} = \text{Sales} - \text{Expenses}$$

Step 2: Next, I set up the equations for sales and expenses.

Let x = number of bottles Let m = cost of making enough concentrate to fill 1 bottle

Each bottle is sold for \$9.72, so: Total Sales = $\$9.72x$

Total Expenses consist of variable expenses and fixed costs.

Variable expenses are those that *vary* according to the number of items made.

Fixed costs are the start-up, or one-time costs.

$$\begin{array}{c} \text{variable expenses} \qquad \qquad \qquad \text{fixed costs} \\ \text{Total Expenses} = (\$1.45x + \$0.05x + (m)x) + (\$65.88 + \$55 + \$26) \\ \begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \text{empty} & \text{label} & \text{cost (\$) of} & \text{mixer} & \text{pans} & \text{kitchenware} & \\ \text{bottle} & & \text{pesticide} & & & & \\ & & \text{concentrate} & & & & \end{array} \end{array}$$

$$\text{Total Expenses} = \$1.50x + mx + \$146.88$$

Step 3: Now, I’ll look for the break-even point in my data or graph. This is the point where there is no gain or loss. In other words, sales equal expenses.

On a graph, it’s the point where the two rays or lines intersect.

Jade’s sketch shows the rays intersecting at 34, so this is the break-even point.

Step 4: I substitute the value of x into my equation to solve for m , the cost of making enough concentrate to fill 1 bottle.

I substitute the break-even point, $x = 34$.

$$\begin{aligned} \text{Sales} &= \text{Expenses} \\ \$9.72(34) &= \$1.50(34) + m(34) + 146.88 \\ \$330.48 &= \$51 + 34m + 146.88 \\ \$330.48 &= 34m + \$197.88 \\ \$132.60 &= 34m \\ \$3.90 &= m \end{aligned}$$

The cost of making just the concentrate is \$3.90/bottle. I have to add in the empty bottle and label, too.

$$\$3.90 + \$1.45 + \$0.05 = \$5.40 \text{ cost to make 1 complete bottle}$$

(continued on next page)

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 8

Although the problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer.

(continued from previous page)

Step 5: When profits are greater than \$500, they will donate 2 bottles for every 15 sold. So, I need to find out how many bottles it takes to get to a profit greater than \$500.

$$\begin{aligned}
 \text{Profit} &> \$500 \\
 \text{Sales} - \text{Expenses} &> \$500 \\
 \$9.72x - [(\$5.40x + \$146.88)] &> \$500 \\
 \$9.72x - \$5.40x - \$146.88 &> \$500 \\
 \$4.32x - \$146.88 &> \$500 \\
 \$4.32x &> \$646.88 \\
 x &> \overline{149.740}
 \end{aligned}$$

Starting with the 150th bottle, they need to donate 2 bottles for every 15 bottles sold.

Step 6: I use another equation to find when they reach a total donation of 26 bottles.

I translate the words into an equation: They donate **26** bottles, which **equals 2 times** the **number of times 15 occurs between** the milestone of **150** bottles sold and the future milestone of an unknown number of bottles sold, **y**. While I'm setting up the equation, I also realize that subtracting 150 from y only provides the difference between those numbers (the number of bottles sold AFTER the 150th bottle). I need to account for the first bottle sold in the club's donation plan (the 150th bottle), so I add the +1 below.

$$\begin{aligned}
 \text{donations} &= 2 \left(\frac{(y-150)+1}{15} \right) & 26 &= 2 \left(\frac{y-149}{15} \right) \\
 & & 13 &= \frac{y-149}{15} \\
 & & 195 &= y - 149 \\
 & & 344 &= y
 \end{aligned}$$

When they've sold the 344th bottle produced, they will have donated 26 bottles to the nursing homes.

$$\begin{aligned}
 \text{Profit} &= \text{Sales} - \text{Expenses} - \text{Sales of 26 bottles} \\
 \text{Profit} &= \$9.72(344) - [\$5.40(344) + 146.88] - \$9.72(26) \\
 \text{Profit} &= \$3,343.68 - [\$1,857.60 + 146.88] - \$252.72 \\
 \text{Profit} &= \$3,343.68 - [\$2,004.48] - \$252.72 \\
 \text{Profit} &= \$1,339.20 - \$252.72 \\
 \text{Profit} &= \$1,086.48
 \end{aligned}$$

(continued on next page)

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 8

Although each problem does have a correct numeric solution, there are multiple pathways students can take to arrive at the answer.

(continued from previous page)

Step 7: Now, I'm ready to find the profit after selling the 344th bottle. Special note: No money was received for 26 of those bottles. I have to subtract that money in my calculations.

Step 8: I must figure out the profit per ounce of solution (not concentrate).

For each 12 oz bottle of concentrate, you need to mix 7 parts water to 1 part concentrate to get the actual amount of solution it will make. I set up and solve a proportion.

$$\frac{12 \text{ ounces con.}}{n \text{ ounces in total}} = \frac{1}{7 + 1} \qquad \frac{12 \text{ ounces con.}}{n \text{ ounces in total}} = \frac{1}{8} \qquad n = 12(8) \qquad n = 96$$

Each 1 bottle of concentrate makes 96 ounces of pesticide solution.

$$\frac{\text{profit}}{\text{ounce}} = \frac{\$1,086.48}{344 \text{ bottles}} * \frac{1 \text{ bottle}}{96 \text{ ounces of solution}} = \frac{\$0.032899709...}{1 \text{ oz solution}}$$

Answer: Their overall profit, per ounce of pesticide solution, when 26 bottles have been donated to the nursing homes, is **about \$0.03 (or 3 cents) per ounce**.