SCHOLASTIC

THE HARDEST MATH PROBLEM STUDENT CONTEST

"One more day!" Sonia thought, stretching after a run. Her smartwatch had shown she'd steadily improved in her triathlon training. (A triathlon is like a marathon with three parts: swimming, biking, and running.)

The sprint triathlon in which she is competing consists of a 750-meter swim, a 20-kilometer bike, and a 5-kilometer run. In her most recent training, Sonia finished the swim in 13 minutes and 45

seconds, the bike ride in 45 minutes, and the run in 19 minutes, each at its own consistent speed.

The next morning, Sonia arrived at the triathlon to find the organizers in a bit of a disarray. "What's going on?!" Baby ducklings were invading the first part of the course. To avoid these cute but ferocious creatures, the organizers set a new starting point farther along the shore, but now the swim consisted of only 425 meters.

GRADE 6

"Hmm...actually this duckling detour is awesome," Sonia thought. She wondered how the change would affect her overall triathlon time. After finishing both her swim and bike ride at her training rate for each, she was amazed to see her overall time. Maybe those ducklings were enough help that she could complete the race at her dream time—if she could manage to speed up her run. **With the duckling detour, she would need to finish her run in _____% of the time of her training run in order to finish the entire race in exactly 70 minutes.** (Give the answer that correctly fills in the blank, rounding to the nearest tenth of a percent.)
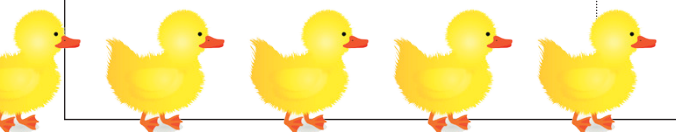
GRADE 7

While Sonia dove right into the water after hearing about the duckling detour, her best friend Stephen was very confused and went to get his questions answered by the organizers. He finally jumped into the water 7 minutes and 23 seconds after Sonia. Stephen's training swim time had been 16 min and 15 seconds, his training bike time had taken 40 minutes, and his training run time was 15 minutes. In training, Stephen had always fallen behind Sonia during their swim and caught back up later in the race. **How many meters into the competition will it take before Stephen catches up to Sonia, assuming they both maintain their training times exactly?**

GRADE 8

Sonia's family cheered for her from the side, except for her younger brother, who cheered wildly for the ducklings. Sonia's brother noticed that when the last competitor jumped in, there were five times as many swimmers as ducklings in the lake (no competitors had finished yet). When Sonia finished swimming, she was the 126th person to leave the water (it was a long race to watch, and his phone had died, so all he could do was count...). There were now twice as many ducklings as swimmers, although the number of ducklings in the lake had not changed during the triathlon. **How many swimmers were in the lake when that last competitor jumped in?**

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CHALLENGE 2 ANSWER KEY

scholastic.com/hardestmathproblem

GRADE 6

Step 1
Convert Sonia's training swim time into seconds:
13 min 45 sec = 825 sec for 750 m

Step 2
Set up two equivalent proportions comparing the old and new swim length distances and times:
750 m / 825 sec = 425 m / x sec

Step 3
Cross multiply or solve algebraically to find that
x = 467.5 seconds.

Step 4
Convert 467.5 seconds to minutes:
= 7 minutes 47.5 seconds is the new swim time

Step 5
Add the new swim time time plus the training times for the bike and run: 7 min 47.5 sec + 45 min + 19 min = 71 min and 47.5 sec; or 4,307.5 sec total

Step 6
Sonia would need to complete the race 1 min 47.5 seconds faster, or 107.5 seconds faster. So the run must be completed in 17 min 12.5 seconds.

Step 7
Set up the proportion to compare the training run time to the new run time needed.
17 min 12.5 sec / 19 min = x % / 100%
Or 1032.5 secs / 1140 secs = x % / 100%

x = 90.57%. Rounding to the nearest tenth of a percent yields 90.6%.

Sonia would need to finish her run in 90.6% of the time of her training run.

Grade 7

Step 1
First, determine how much of a lead Sonia has after the swim.
Sonia's training time was 13 min 45 sec = 13.75 min, or 825 seconds for 750 m.

Find Sonia's new swim length time:
750 m / 825 secs = 425 m / x secs
x = 467.5 seconds
Convert 467.5 seconds to minutes: 467.5 / 60 = 7.79167 minutes.
That's 7 minutes and .79167 of a minute.
.79167 * 60 seconds = 47.496 seconds
= 7 minutes 47.5 seconds for Sonia's new swim length time

Stephen's training swim time was 16 min 15 sec for 750 m.
750 m / 975 secs = 425 m / x secs
x = 552.5 seconds, which is **9 min 12.5 secs**

After the swim, Stephen's time is at 7 min 23 sec + 9 min 12.5 sec = 16 min 35.5 sec. Subtracting Sonia's time of 7 min 47.5 sec from Stephen's time, **Sonia has a lead of 8 min 48 sec after the swim.**

Step 2
Recognize that Stephen will not catch up during bike race...
For Sonia: 20 km took 45 min
For Stephen: 20 km took 40 min

Stephen's biking was 5 minutes faster than Sonia's. Her lead of 8 min and 48 seconds, minus those 5 minutes, means Sonia's lead is now 3 min 48 sec (or 3.8 min lead).

(*Grade 7 continued*)

Step 3

Finding unit rates:

Sonia's running rate is 3.8 min per km (derived from 19 min/5 km), and Stephen's rate is 3 min per km (derived from 15 min/5 km).

With Sonia's 3.8 min lead, set up a proportion showing when her time will equalize with Stephen's:

$$3.8x \text{ (min/km)} = 3.0x \text{ (min/km)} + 3.8 \text{ min}$$
$$-3.0x \qquad\qquad -3.0x$$
$$0.8x \text{ (min/km)} = \qquad\qquad 3.8 \text{ min}$$
$$x = 4.75 \text{ km}$$

Stephen will catch up after running for 4.75 km.

Step 4

Recognize the total distance traveled over the course of the triathlon is actually:

425 m (which is .425 km) + 20 km + 4.75 km = 25.175 km

Step 5

Recognize the final answer must be in meters.

25.175 km converted to meters is 25,175 meters.

It will take 25,175 meters for Stephen to catch up to Sonia during the triathlon.

Grade 8

Step 1

The swimmers outnumbered the ducklings by five times in the lake.

> Let w = swimmers
> Let d = ducklings
> $w = 5d$

Step 2

When Sonia finished swimming, she was the 126th person to leave the water. There were now twice as many ducklings as swimmers, which can be represented as:

> $2(w - 126) = d$

Step 3

Recognize therefore that $2(5d - 126) = d$

> $10d - 252 = d$
> $9d = 252$
> $d = 28$ ducklings

If 5 times as many ducklings as swimmers were in the lake, then **there were 140 swimmers in the lake when the last competitor jumped in**.