### Geometry Works! The Stage Takes Shape

The Geometrics want to play a special concert at your school, but they need a stage that’s just right. The drummer wants an elevated trapezoid-shaped platform. The bass player wants to be on a raised triangular prism. The guitarist, a sphere. The keyboard player wants a circle. The singer, a square pyramid.

To design the Geometrics Stage, the crew must calculate the surface area and volume of different shaped sections. The drummer wants an elevated trapezoid-shaped platform. The bass player wants to be on a raised triangular prism. The guitarist, a sphere. The keyboard player wants a circle. The singer, a square pyramid.

#### The drummer wants to have a raised trapezoid-shaped platform. She needs the Geometrics Stage team to help her.
- **Base:** 10 feet, **height:** 6 feet, **base 1:** 8 feet, **base 2:** 5 feet
- **Height:** 6 feet

The total area of the platform is 55 square feet. The drummer is happy with the total area of the platform. The bass player asks the Geometrics Stage team to help him too. He needs a raised triangular prism. The length of the triangular base is 8 feet, the width is 6 feet, and the slant height is 7.8 feet.

The total area of the triangular prism is 43 square feet. The bass player is happy with the total area of the triangular prism. The guitarist, a sphere. The height of the sphere is 7 feet. The total area of the sphere is 61 square feet. The guitarist is happy with the total area of the sphere. The keyboard player wants a circle. The keyboard player asks the Geometrics Stage team to help her. She needs a raised circle. The radius of the circle is 3.5 feet. The total area of the circle is 31 square feet. The keyboard player is happy with the total area of the circle. The singer, a square pyramid. The base length is 4 feet, the base width is 4 feet, and the slant height is 7.28 feet. The total area of the square pyramid is 25 square feet. The singer is happy with the total area of the square pyramid.

### Perimeter, Area, Surface Area, and Volume: Review of Terminology, Basic Shapes, and Formulas

- **Perimeter:** The distance around the edge of a circle
- **Area:** The measure of a bounded region of a two-dimensional figure
- **Surface Area:** The total distance around the outside of a three-dimensional figure
- **Volume:** The measure of space a three-dimensional figure takes up

#### 2D Shapes and Formulas

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>2(l + w)</td>
<td>l × w</td>
</tr>
<tr>
<td>Square</td>
<td>4s</td>
<td>s²</td>
</tr>
<tr>
<td>Triangle</td>
<td>a + b + c</td>
<td>½ab sin C</td>
</tr>
<tr>
<td>Circle</td>
<td>2πr</td>
<td>πr²</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>(a + b)l</td>
<td>½(a + b)h</td>
</tr>
</tbody>
</table>

#### 3D Shapes and Formulas

<table>
<thead>
<tr>
<th>Shape</th>
<th>Surface Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Prism</td>
<td>2hl + 2l(w + h)</td>
<td>lwh</td>
</tr>
<tr>
<td>Square Pyramid</td>
<td>½pl + 2b²</td>
<td>½b²h</td>
</tr>
<tr>
<td>Cylinder</td>
<td>2πrh + πr²</td>
<td>πr²h</td>
</tr>
<tr>
<td>Sphere</td>
<td>4πr²</td>
<td>⅖πr³</td>
</tr>
</tbody>
</table>

**Notes:**
- Perimeter is the distance around the edge of a shape.
- Area is the measure of the space inside a shape.
- Surface Area is the total area of all surfaces of a three-dimensional figure.
- Volume is the measure of space a three-dimensional figure takes up.

#### Abbreviations:
- C = circumference
- d = diameter
- h = height
- A = area
- V = volume
- P = perimeter
- SA = surface area

### Pack It Up! What Will Fit?

The Geometrics Stage crew has to transport the painted stage by using a rectangular prism, a cylinder, and a square pyramid to the concert. They have to make sure that the objects do not touch the stage. The height of the concert is 8 feet, the width is 5 feet, and the depth is 13 feet. They need to make sure that the volume of the stage is not too large and can be transported by the van. The volume of the stage is 400 cubic feet. They will need to divide the volume of each shape by the volume of the stage to see how much space each shape will take up.

#### The Dimensions of the Shapes Used

- **Rectangular Prism:** length = 4 feet, width = 3 feet, and height = 8 feet
- **Sphere:** radius = 2 feet
- **Cone:** slant height = 7.8 feet, and height = 3 feet

**Notes:**
- The dimensions of the rectangular prism, the sphere, and the cone are the same height and radius as the cylinder and have the same slant height. The dimensions of the cylinder and cone are also the same height. The dimensions of the rectangular prism are 3 feet, 1 foot, and 4 feet.

#### Surface Area and Volume

- **Surface Area:** The total area of all surfaces of a three-dimensional figure.
- **Volume:** The measure of space a three-dimensional figure takes up.

#### Problem Set

1. **Cone:** The slant height of the cone is 5 feet. The radius of the base is 3 feet. The volume of the cone is calculated using the formula ⅓πr²h.
2. **Sphere:** The radius of the sphere is 2 feet. The volume of the sphere is calculated using the formula ⅖πr³.
3. **Cylinder:** The height of the cylinder is 6 feet. The radius of the base is 3 feet. The volume of the cylinder is calculated using the formula πr²h.

**Notes:**
- The dimensions of the rectangular prism and the cone are the same height and radius as the cylinder.
- The dimensions of the sphere are twice as large as the dimensions of the rectangular prism.
- The dimensions of the cone are one-third the dimensions of the sphere.

**Conclusion:**
- The dimensions of the cone, sphere, and cylinder are all proportional to each other.
- The volume of the cone is ⅓ of the volume of the sphere, and the volume of the cylinder is twice the volume of the sphere.

**Bonus:**
- The volume of the cone is 106 cubic feet. The volume of the sphere is 35 cubic feet. The volume of the cylinder is 113 cubic feet.
- The total volume of all the objects combined is 254 cubic feet.

**Notes:**
- The total volume of all the objects combined is smaller than the volume of the stage.
- The objects will fit in the van.