Development of Math Concepts Age by Age

	0–3 years	4 years	5 years	6-7 years
General Development	Children may:	Children may:	Children may:	Children may:
Forming Concepts	learn concepts in action	learn concepts in an example-by-example way	learn concepts through a particular example	more easily learn con- cepts that are thought of in terms of rules
Representing and Symbolizing	at age 2, begin to develop mental representations, including symbols			represent and mentally "undo" a process
NUMBERS				
Number Concepts	recognize very small numbers, nonverbally, and then with numerical labels, such as <i>two</i> ; at about age 2, represent numbers exactly; begin to use the <i>stable-order rule</i> , and even the <i>abstraction rule</i> , in counting small collections	maintain the <i>one-to-one</i> rule in counting increasingly large collections; understand the cardinal rule (the last number word in counting tells how many are in the collection)	begin to count, not just discrete objects, but classes, such as how many different colors of blocks there are, and units, such as how many whole eggs, when some halves are together and some are not; begin to understand the implica-	tions of the <i>order-irrele-</i> vance rule
Comparing Numbers	visually determine whether very small col- lections have the same amount, or which has more	use counting or matching to compare two collec- tions of up to five objects, despite deceptive appear- ances	use counting to compare two collections, even if the objects they contain are a mixture of sizes and types	use counting to accurate- ly compare two collec- tions, even if the collec- tion with the smaller number has objects that are larger in size
Adding and Subtracting	recognize how many objects should be present when one is added or taken away from a very small collection	solve word problems using objects, with sums of up to five	solve word problems using counting-based strategies; for example, when asked, "If you had four toys and got two more toys, how many would you have?" will count four fingers, then count up with two more	fingers
GEOMETRY AND MAPS				
Shapes	match simple shapes	recognize and name varia- tions of the circle, square, triangle, and rectangle	recognize and name shapes in various orienta- tions, sizes, and types; start to recognize the parts of shapes, such as sides and angles	sort shapes into classes based on their attributes, such as triangles' having three straight sides
Maps	understand and use ideas such as over, under, above, on, beside, next to, between	build a simple but mean- ingful map with landscape toys, such as houses, cars, and trees; learn a simple route from a map	place toy objects in the correct relative position to make a map of the class- room	make and follow maps of familiar areas, using some measurements
PATTERNS AND ALGEBRA				
Patterns and the Number Patterns Leading to Algebra	act out patterns, such as jumping to the left, right, left, right; observe repeating patterns, such as a block standing, then lying down, then standing	copy simple repeating patterns, such as ABBAB- BABB	separate the "core unit" in patterns, such as ABA in ABAABAABA; find pat- terns in math, i.e., adding one to a number results in the next "counting num- ber"	create, recognize, and use early algebraic patterns; for example, subtracting a number from itself gives you zero, or $n-n=0$