

## ANSWER KEY

### Baseball Stadium

#### Seats

**Q:** The stadium's seating options will include premium seats (boxes closest to the action), medium-priced seats, and budget-friendly bleachers. Your design calls for 4,500 upper-deck premium seats and 7,500 lower-deck premium seats. The team's policy is to have a 3:2 ratio of premium seats to bleacher seats. How many bleacher seats will the stadium include?

**A:** 8,000 bleacher seats.  $4,500 + 7,500$  premium seats = 12,000.  $12,000/x = 3/2$ .  $x = 8,000$ .

#### Stairs

**Q:** The stairs must be wide enough for fans to come and go safely. But wider stairs mean fewer seats. To maximize revenue by having as many seats as is safe, you design 6-foot-wide stairs for each section of 30 seats across. A seat is 24 inches wide. What is the simplified ratio of the width of a seating section (yields revenue) to the width of the stairs (no revenue)?

**A:** 10:1. 24 inch seat = 2 feet.  $2 \times 30$  seats = 60-foot section.  $60:6$  simplified = 10:1

#### Lights

**Q:** Your stadium will have 8 large light towers to illuminate the field for night games. In your blueprints, the towers are 9 inches tall. If you are using a scale of 1:220 (1 foot in the drawing equals 220 feet in the real world), how many feet tall will each light tower be when it is built?

**A:** 165 feet. First, convert 9 inches to 0.75 feet.  $0.75 \times 220 = 165$  feet

#### Grass

**Q:** According to your blueprints, the field will measure 120,000 square feet. The dirt infield will take up  $1/10$  of the square footage of the field. Your groundskeeper estimates that she needs to seed the grassy areas of the field at a ratio of 1 pound of grass seed for every 4,000 square feet. How many pounds of grass seed will she need to do the job?

**A:** 27 lbs. If whole field seeded:  $120,000/4,000 = 30$  lbs. But only  $9/10$  of field is grass:  $30 \times 9/10 = 27$ .

#### Scoreboard

**Q:** Your design includes a state-of-the-art scoreboard in center field measuring 60 feet high by 150 feet wide. Fans sitting in center field can't see this scoreboard, so your design includes 2 auxiliary scoreboards in the infield. If you want to keep the dimensions in the same ratio, and each auxiliary scoreboard will be 20 feet high, how wide will it be?

**A:** 50 feet. Set up a proportion of the dimensions to find an equivalent ratio:  $60/150 = 20/x$ .  $150 \times 20 \div 60 = 50$ .

### **Bonus Questions**

- 1) The stadium will have 3 circular concourses surrounding the seating areas on each level. These are the perfect spots for food and souvenir stands. Team management has determined that a 7:2 ratio of food stands to souvenir stands will help keep lines shorter. If your stadium design includes 42 food stands in total, how many souvenir stands will you include in your plan for **each** concourse level?

**Answer:** 4 souvenir stands. Food stands per level:  $42 \div 3 = 14$ . Use the ratio of food to souvenir stands to set up the proportion  $7/2 = 14/x$ .  $x = 4$ .

- 2) Many of your team's fans love giveaway promotions. This year, you will be giving away bobblehead dolls of the team's leading homerun hitter to fans 14 years of age and younger. If the team expects a sellout of 52,000 fans, and, historically, there has been a 7:6 ratio of fans 14 and under to fans over 14, how many bobbleheads will the team give out?

**Answer:** 28,000 bobbleheads. A 7:6 ratio means that  $7/13$  of the crowd will be 14 or under.  $7/13 \times 52,000 = 28,000$ .

- 3) The team's souvenir department includes special designer clothing, which made up 1% of the team's total clothing sales of \$20,000,000. The manager says that with more promotion, the ratio of designer to non-designer clothing sales can be 1:9. If total clothing sales are expected to be to \$22,000,000 next year, by how many dollars must designer clothing sales increase to meet the ratio goal (if non-designer sales stay the same)?

**Answer:** \$2,000,000. 1:9 ratio means 10 equal parts total. Total sales of \$22,000,000  $\div$  10 = \$2,200,000 per part. Subtracting current sales of \$200,000 from the goal of \$2,200,000 means sales must increase by \$2,000,000.

### **"Think About It" Questions**

**Question 1:** The stadium design currently has a 3:2 ratio of premium seats to bleacher seats (12,000 premium seats and 8,000 bleacher seats). If the team wants to keep the total of premium seats and bleacher seats at 20,000 but wants to change the ratio to 4:1, how many of each seat would the stadium have? Furthermore, what consequences could result from building the stadium with this new ratio?

**Answer 1:** After moving to a 4:1 ratio, the stadium would have 16,000 premium seats and 4,000 bleacher seats ( $20,000 \div 5 = 4,000$  bleacher seats, and  $20,000 - 4,000 = 16,000$  premium seats). The consequences of this change might include:

- Increased revenue, **if** there is sufficient demand for premium seating to fill the seats with paying customers
- Decreased revenue, **if** the premium seats go unsold
- Increased costs to upgrade the seating. Fans paying premium prices will want more comfortable seats and a more attractive stadium section.
- Potential loss of fans in the long run if people on a budget or families with children are unable to afford the increased cost of tickets

**Question 2:** One of your fellow designers came up with an alternative design for the light towers. Instead of 8 towers, each 165 feet high, he suggested 1 tower 1,320 feet high. He reasoned that everything stays in proportion since  $8/1 = 1,320/165$ , and it would make a brilliant architectural statement. What do you think of your colleague's reasoning?

**Answer 2:** Although the calculations are correct, your colleague's idea would be a disaster, unless the team wanted to stop playing night games! The field will receive a lot less illumination from one light tower if it is moved 8 times further from the field. Reducing the number of towers from 8 to one would make the field even darker, and what light there is would be concentrated in one area. Just because a calculation is accurate doesn't mean it should be followed without thinking. Though one 1,320-foot tower is as long as 8 165-foot towers, that doesn't mean that one 1,320-foot tower would be a better alternative to 8 165-foot towers because the single tower doesn't provide enough light.

**Question 3:** The team has a goal of having a 1:9 ratio of designer clothing sales to non-designer clothing sales. Assuming there are no other categories of clothing, what fraction of total clothing sales would designer clothing consist of? What percentage? In general, what do ratios have in common with fractions and percentages, and how do they differ? Which representation do you think is easiest to work with? Can you think of circumstances where one representation would be better than the other?

**Answer 3:** A ratio represents a part-to-part relationship while a fraction represents a part-to-whole relationship. So if there is a 1:9 ratio, then the total consists of 1 part designer clothing and 9 parts non-designer clothing. The fraction consisting of designer clothing to total clothing equals  $1/(9+1)$  or  $1/10$ . As a percentage,  $1 \div 10 = 0.1 = 10\%$ .

Ratios, fractions, and percentages require one to identify what are the parts and what is the whole. Ratios and fractions both consist of 2 numbers, but a ratio is a comparison of 2 parts, whereas a fraction depicts one part compared to the whole. In contrast, a percentage is made up of one number, but the percentage sign means that the number is part of 100 (percent means “out of 100”), so the number can be understood to be the numerator of a fraction with a denominator of 100.

In some cases, there is no real advantage to using one form over the other, e.g., “girls make up  $\frac{5}{9}$  (56%) of the class” versus “there is a 5:4 ratio of girls to boys in the class.” In other cases, say for scale on a map, it is easier to understand a ratio of 1 inch:1 yard than being told that distance on the map is  $\frac{1}{36}$  of the distance in the real world.

## **Amusement Park**

### **Roller Coaster**

**Q:** The park’s main attraction will be the roller coaster *Mountain Scream*, which will be designed to be fast and fun, but still secure and safe. *Mountain Scream*’s manufacturer recently built a coaster in England that used 2,500 tons of steel and 50,000 bolts. If *Mountain Scream* will use 500 more tons of steel than the English coaster, how many bolts will be needed?

**A:** 60,000 bolts. Set up the proportion  $\frac{50,000}{2,500} = \frac{x}{3,000}$ .  $x = 60,000$ .

### **Coaster Cars**

**Q:** *Mountain Scream*’s train of cars will pick up speed by gaining kinetic energy from a large hill. The cars will then rocket through 3 loops in the track, starting with a heart-stopping loop 180 feet high, another loop 90 feet high, and culminating in a loop 60 feet high. In simplest terms, what is the ratio of the heights of the 3 loops the cars will make?

**A:** 6:3:2. The unsimplified ratio is 180:90:60. Dividing each part of the ratio by the common factor 30 equals 6:3:2.

### **Ferris Wheel**

**Q:** Your plan includes the Ferris wheel *Whirlwind*, a ride that promises classic fun at breakneck speed. Managing wait time while people are loaded into its 16 cars will be challenging. It’s possible to load 2 cars at a time, which takes 45 seconds. The ride begins when all cars are full. You want a 3:2 ratio of ride time to load time. How long should the ride be?

**A:** 9 minutes. Loading:  $\frac{45}{2} = \frac{x}{16}$ .  $x = 360$  seconds, or 6 minutes. Ride:  $\frac{6}{2} = \frac{x}{3}$ .  $x = 9$  minutes.

### **Drop Tower**

**Q:** A must-have for any thrill seeker is a drop tower ride. The *Precipice Plummet* will stand 420 feet tall. The scale in your architectural diagrams is 1":40'. How tall will your drawing of *Precipice Plummet* be?

**A:** 10.5 inches tall.  $420/40 = x/1$ , where  $x$  equals the height of the drawing.  $420/40 = 10.5$ .

### **Food**

**Q:** A well-designed park plans for customers' dining needs. "Grab-and-go" food carts and food stands with a more varied menu are a significant source of revenue. The park's 2 food stands are projected to have revenues of \$900,000 each. 3 carts are planned. Management expects a 4:1 ratio of food stand revenue to cart revenue. How much revenue will the average cart have?

**A:** \$150,000.  $\$900,000/1 = x/2$ .  $x = \$1,800,000$ .  $1,800,000/4 = x/1$ .  $x = \$450,000$ .  $450,000/3 = x/1$ .  $x = \$150,000$ .

### **Bonus Questions**

- 1) Adrenaline Point's owners envision a park where even the snacks give thrills. For the park's signature snack, the owners have decided to create "Caution Candy," a spicy-hot cotton candy concoction. Cotton candy is 99.999% spun sugar with traces of flavoring and artificial color. What is the ratio of non-sugar ingredients to sugar in a serving of cotton candy (in whole numbers)?

**Answer:** 1:99,999. Set up the percentage of non-sugar on the left side of the colon and the percentage of sugar on the other: .001:99.999. Convert to whole numbers by multiplying both sides by 1,000: 1:99,999.

- 2) A "rule of thumb" in the amusement park industry is that owners should plan to spend around \$110 in development costs for every guest expected in the first year. Since the owners of Adrenaline Point want their park to be top of the line to encourage repeat visits, they expect to spend 50% more than the average per-visitor amount. If they spend \$412,500,000 to develop the park, how many first year visitors do they expect?

**Answer:** 2,500,000 visitors. First, find the development costs per expected visitor by setting up a proportion  $110/100 = x/150$ .  $x = \$165$ . To find the number of first-year visitors, set up  $1/165 = x/412,500,000$ .  $x = 2,500,000$

- 3) One design challenge you face is to make the rides *seem* risky but actually be quite safe. Though the probability of serious injury at a U.S. amusement park is only 1 in 24 *million*, the chance of an accident still exists. So, the park's owners purchased insurance and found they would pay \$3 for every \$1,000 in revenue annually. If the park budgets \$750,000 for insurance for the first year, how much revenue is expected?

**Answer:** \$250,000,000. Set up a proportion  $1,000/3 = x/750,000$ .  $x = \$250,000,000$ .

### ***“Think About It” Questions***

**Question 1:** Adrenaline Point wants to offer a premium Ferris wheel experience for a premium price. The owners are thinking of charging 3 times the normal fare for a longer ride. If load time for the ride equals 9 minutes and management wants to increase the current 13.5 minute ride time by 100%, what will the ratio of ride time to load time be in simplest terms?

**Answer 1:** 3:1. Increasing ride time by 100% means that the new ride time will be twice the old time.  $13.5/1 = x/2$ . Solving for  $x$ , the new ride time,  $x = 27$ . The ratio of ride time to load time is 27:9. In simplest terms, this is 3:1.

**Question 2:** Your design for Adrenaline Point includes a gift shop of 5,000 square feet. Park management expects sales of \$1,400 per square foot. You would like to use 1,250 square feet for a museum-quality art exhibit to make the shop a special destination in the park.

- What would the shop’s revenue be with the exhibit, assuming the revenue estimate of \$1,400 per square foot is correct?
- What would revenue be if the shop used all 5,000 square feet for sales space?
- Do you think the shop should use the space for the exhibit or not?

**Answer 2:**

- Revenue with the exhibit would be \$7,000,000. Set up a proportion  $1,400/1 = x/5,000$ . Solving for  $x$ , revenue equals \$7,000,000.
- Revenue without the exhibit would be \$5,250,000. Set up a proportion  $1,400/1 = x/(5,000 - 1,250)$ . Solving for  $x$ , revenue equals \$5,250,000.
- Since the future is unknowable, the question of whether the shop should include the exhibit space or not has no right answer. Devoting space to the exhibit appears to cost \$1,750,000 in lost revenue, but if the exhibit attracts more customers, it is possible that revenue might exceed the \$1,400 per square foot estimate.

**Question 3:** Diablo’s Domain had revenue of \$250,000,000 in its first year as expected. The company paid \$750,000 for insurance or \$3 for every \$1,000 in revenue. If the park is wildly successful and revenue increases by 25% in the second year and the rate of \$3 for every \$1,000 holds true, how much would it pay for insurance? Do you think it’s reasonable for insurance costs to increase proportionally?

**Answer 3:** Insurance costs would amount to \$937,500. Set up the proportion  $750,000/1 = x/125$ . Solving for  $x$ , insurance costs in the second year would amount to \$937,500. Since the number of serious accidents is essentially proportional to the number of customers going on rides, the risk to the insurance company would increase as the number of customers increases.

## Aquarium Research Center

### Tank

**Q:** You are designing a new, larger coral reef exhibit that will stun visitors with its size and vibrant colors. The tank requires special glass to protect this delicate environment. The glass costs \$25,000 + a 10% engineering fee + a 5% shipping fee calculated on the cost of the glass plus engineering charges. What is the total cost of the glass, including fees?

**A:** \$28,875.  $10/100 = x/25,000$ .  $x = \$2,500$ .  $5/100 = x/(\$25,000 + \$2,500)$ .  $x = \$1,375$ .  $\$27,500 + \$1,375 = \$28,875$ .

### Water:

**Q:** The new, larger coral reef tank will thrill visitors by providing a floor-to-ceiling view of sea creatures swimming through the reef. In the old version of the exhibit, 576,000 gallons of water circulated through the tank over 24 hours. If the new exhibit is  $3\frac{1}{2}$  times larger than the old one, how many gallons per hour will be circulated?

**A:** 84,000 gallons per hour.  $1/24 = x/576,000$ .  $x = 24,000$  gallons per hour.  $3.5/1 = x/24,000$ .  $x = 84,000$ .

### Coral Reef:

**Q:** A coral reef consists of millions of individual coral organisms in colonies. The exhibit's reef will grow at a rate of about  $3/4$  of an inch per year. In your design, the coral reef is 3 meters from the glass. You want to check if you need to worry about the growing coral breaking through the glass. How many years will that take? **Note:** 1 inch = approx. 0.025 meters.

**A:** 160 years.  $3/4$  inches/year = .01875 meters/year ( $.025/1 = x/(3/4)$ ).  $1/.01875 = x/3$ .  $x = 160$  years.

### Fish:

**Q:** The Living Ocean Institute is almost done purchasing fish species to populate the coral reef habitat. Spectacular yellow striped butterflyfish, likely to wow visitors, are available from an exotic fish supplier for \$2,480 for 4 fish. If the Institute wants to purchase 15 butterflyfish, how much will the butterflyfish cost in total?

**A:** \$9,300. Cost per fish =  $\$2,480/4 = \$620$ . Cost for 15 fish =  $\$620/1 = x/15$ .  $x = \$9,300$ .

### Clownfish:

**Q:** In the wild, colorful orange and white clownfish eat algae and small crustaceans. In an aquarium, clownfish are typically fed finely chopped shrimp. The marine biologist knows that 15 clownfish would receive 300 servings of shrimp over a 5-day period. If 22 clownfish are added to the exhibit, how many servings of chopped shrimp will the fish eat in a week?

**A:** 616 servings. Servings/day, 15 fish:  $300/(5 \times 15) = 4$ . Servings/week, 22 fish:  $4/1 = x/(22 \times 7)$ .  $x = 616$ .

### **Bonus Questions**

- 1) The Living Ocean Institute owns 2 research vessels that will be docked next to the new building. The *Delfino* recently took 5 hours to cover 57.5 miles of shoreline. On its last research voyage, the *Balina* surveyed 54 miles in 4.5 hours. Which vessel is faster?

**Answer:** The *Balina* is faster. For the *Delfino*,  $57.5 \div 5 = 11.5$  miles per hour, while for the *Balina*,  $54 \div 4.5 = 12$  miles per hour.

- 2) The Living Ocean Institute is well known for the care it provides to birds harmed in environmental disasters like oil spills. If an average of 10 birds per month were treated this year and a total of 100 birds were treated the year before that, by what percentage did the number of birds treated per year increase?

**Answer:** 20%. 10 birds per month in the current year equal an annual rate of 120 birds. The percentage increase equals the amount of the increase  $(120 - 100 = 20) \div 100 = 20\%$ .

- 3) Visitors love watching the sea otters come right up near to the deck to feed on crabs, mussels, and other marine species. The average otter needs to eat food equal to 25% of its weight every day. If the average male otter weighs 65 pounds and the average female weighs 45 pounds, how much more food does a male otter need each day than a female otter?

**Answer:** 5 pounds more per day. Set up a proportion to find the male otter's daily food needs:  $65/1 = x/4$ .  $x = 16.25$  pounds. Then, find the female otter's daily food needs:  $45/1 = x/4$ .  $x = 11.25$  pounds.  $16.25 - 11.25 = 5$  pounds more.

### **"Think About It" Questions**



**Question 1:** The Living Ocean Institute would like to modernize their cafeteria to increase use and raise funds for the Institute's research projects. You designed a calming dining space in aquatic blue and green tones. To keep costs down, the Institute has indicated that one cashier should be enough. In their current cafeteria, they serve 120 meals a day during their operating hours of 10 a.m. to 4 p.m. A cashier can ring up an order in 90 seconds. Do you think that one cashier is enough? Why or why not? Can you foresee any problems that might result?

**Answer 1:** The cashier can process 40 customers per hour ( $90 \text{ seconds} / 60 \text{ seconds} = x/1$  so  $90 \text{ seconds} = 1.5 \text{ minutes}$ , and  $60 \text{ minutes in an hour} / 1.5 \text{ minutes per customer} = x/1 \text{ hour}$  so they can process 40 customers per hour). They can process 240 customers over a six-hour day ( $40/1 = x/6$ , so  $x = 240$ ). Since a cashier can handle 240 customers in a six-hour day and they are only serving 120 meals per day, there would seem to be plenty of excess capacity. However, this assumes that customers come at a constant rate throughout the day. So at peak periods (probably 11:30 a.m. – 1:30 p.m. for lunch) the rate might be double the average hourly rate and there could be significantly long waits for the cashier. Additionally, your striking design for the new facility should bring in even bigger crowds, and unhappy paying guests who have a long wait for the cashier might not come back to visit a second time.

**Question 2:** The Living Ocean Institute is an environmentally responsible organization, and your design includes solar panels on the roof to supply all the building's power requirements. The sales representative from the solar panel company tells you that the total retail cost is \$90,000, but that the company offers a 10% discount as a sales promotion, plus a 5% discount for full payment within 30 days. The sales representative claims the Living Ocean Institute will owe a total of \$76,500 with both discounts taken. Is the sales representative correct? Explain your answer.

**Answer 2:** The sales representative is not correct. Some students will make the mistake of adding 10% and 5% for a total discount of 15%.  $15/100 = x/90,000$  does equal a discount of \$13,500, and  $\$90,000 - \$13,500$  does equal \$76,500, but the cash discount should be applied **after** the promotional discount is applied. So  $10/100 = x/90,000$  equals a promotional discount of \$9,000, leaving a net cost of \$81,000 ( $\$90,000 - \$9,000$ ). Applying the 5% discount to \$81,000 ( $5/100 = x/81,000$ ) gives a discount of \$4,050, so the net price of the panels is \$76,950 ( $\$81,000 - \$4,050$ ).

**Question 3:** The Institute prides itself on the educational programs it offers local students in its environmental classroom. It expects to staff the classroom with 2 scientists per field trip. To maintain a good learning environment, the Institute wants no more than 36 students per field trip and wants to schedule 4 field trips per day. Part of each field trip includes looking at a

sample of seawater through a high-powered microscope to observe tiny organisms. Your design includes lab tables with 3 microscope stations per table, and a rate of 3 students per microscope is appropriate. How many lab tables should your design include? Explain how you determined the number of lab tables.

**Answer 3:** 4 lab tables. There is a lot of unit rate information in this problem and the trick is for the students to sort out the relevant from the irrelevant. In this case, all that matters is that the classroom needs to accommodate 36 students, one microscope per 3 students, and 3 microscopes per table. The parts of the problem can be worked on in a different order but one way is to find the number of students per table by setting up a proportion:  $3/1 = x/3$ , that is, 3 students per microscope to  $x$  students per 3 microscopes, the number of scopes that can fit at a table. 9 students can fit at a table.  $1/9 = x/36$ , i.e., if one table is needed for 9 students, how many tables are needed for 36? The answer is 4 tables.